# Numerical prediction of turbulent heat transfer in gas pipe flows subject to combined convection and radiation

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An analysis is made of the interactive heat transfer problem involving turbulent forced convection and radiation in the thermal development region of a gas pipe flow. A source of distortion of temperature and heat transfer rates is usually attributed to the presence of thermal radiation in high-temperature gas flows. In particular, this paper is concerned with a situation wherein an absorbing-emitting gas having a fully developed turbulent velocity enters an isothermal pipe with black walls. The turbulent model adopted for the velocity profile involves the solution of one differential equation for the kinetic energy of turbulence. Under the idealization of gray gas, the radiation contribution is modeled by a differential method, the so-called method of moments, that circumvents the partial integrodifferential equation typical of this kind of problem. Accordingly, the new formulation governing the combined heat exchange process accounts for a coupled system consisting of a partial differential equation for temperature and an ordinary differential equation for the irradiation. The former is solved by a hybrid methodology using the method of lines in conjunction with a control volume discretization in the radial direction only. Similarly, the latter is discretized by control volumes too. Solutions of the resulting initial value problem were obtained numerically by a Runge-Kutta-Fehlberg scheme, which deals successively with the associated system of algebraic equations. Remarkably rapid convergence was achieved by adopting a coordinate transformation that clusters grid points near the wall. Results based on 10 lines are presented for the axial distributions of bulk temperature and total Nusselt numbers as a function of the controlling parameters of the combined heat exchange process. The numerical predictions have been compared with the available results, and the agreement was satisfactory for all cases tested.

**Keywords:** turbulent forced convection; radiation; pipe flow

## Introduction

Transfer of heat by simultaneous convective and radiative transfer at high temperatures and high heat fluxes has become increasingly important in the analysis and design of high-temperature gas-cooled nuclear reactors (HTGR), advanced energy conversion devices, furnaces, combustors, etc. These, and many other applications, have provided the impetus for research on radiation transfer and on combined convection radiation in participating media. Further information is given in a recent survey by Viskanta.[1] In such applications, momentum, energy and radiation transport equations must be solved simultaneously in order to determine local temperatures and heat transfer rates in the media, unless radiation is either very strong or very weak. Under these extreme conditions, either the dominant mode only need to be considered or the minor flux can be superposed onto the major flux. However, in flow applications where radiation and turbulent convection are coprincipal heat transfer mechanisms, coupling between these different models of energy transfer is unavoidable, and consequently the descriptive equations are interlinked.

Accounting for the influence of radiation on the local heat transfer in the thermal entrance region of ducts is a difficult problem. The difficulties are mainly related to the transport processes of the controlling mechanisms of radiation and turbulent convection. In the former, the energy balance is usually expressed in terms of a nonlinear integrodifferential equation, while in the latter, suitable turbulence models have to be incorporated into the energy equation.

Nichols[2] studied exclusively the influence of absorption of radiation on the temperature profile and heat transfer to a medium flowing turbulently in an annulus. Landram *et al.*[3] and Larsen *et al.*[4] treated the thermally developed region in turbulent pipe flow invoking optically thin gas radiation. The analysis of Ref. 4 includes some experimental data for water vapor also. Tsou and Kang[5] analyzed the turbulent flow of a radiating gas as a conjugate problem in the fluid domain. These authors allowed for the propagation of axial radiation upstream of the heat exchange section.

A number of analyses dealing with internal gas flows have assumed fully developed temperature distribution, which reduces the partial integrodifferential equation of energy to an ordinary integrodifferential equation. However, Kurosaki[6] and, more recently, Chawla and Chan[7] have found that, under strict laminar conditions, total Nusselt numbers increase downstream of the position of minimum rather than approach an asymptotic value as in the case of pure convection. This peculiarity is particularly notorious when radiation is strong compared to conduction.

The complexity of turbulent radiating flows suggests that the gray gas condition be studied first. According to Tien[8], investigations of this simple model for flow in tubes results in great insight, providing first-order approximations and also increased understanding of more general phenomena. For this reason, an analysis of the turbulent heat exchange process in the entrance region of a pipe, assuming gray conditions, was undertaken. Conversely, one of the main problems in dealing with combined forced convection and radiation is the large computation time needed for a reliable prediction of temperature distributions in the gas flow. Thus, it becomes important to seek alternative formulations that may simplify the radiation transport equation. In view of this, various approximate differential formulations have been developed in the past. Among these, the spherical harmonics, the method of moments, the multiple flux model, and the discrete ordinate method provide means to obtain higher-order approximate solutions to the equation of radiative transport (see Ozisik[9]). Additionally, it has been shown by Krook[10] that the first moment method and the P1 approximation of the spherical harmonics are entirely synonymous. This similarity is equally valid for the two-flux model in one-dimensional cases. Therefore, this equivalent approach is being adopted in the present study, which results in a conventional partial differential equation for energy accounting for a source term that depends on both temperature and irradiation. This equation is coupled to a rather simple ordinary differential equation where the dependent variable (the radial variable) is the irradiation itself. The hydrodynamic aspects of the problem have been handled by the $(K, L)$-model of turbuence reported by Rodi.[11] The use of this model furnishes the fully developed velocity profile, which is an input for the energy equation.

Although a wide variety of numerical techniques could be used to solve the problem in question, one that is particularly well suited for solving the descriptive system of partial differential equations involving two independent variables is the method of lines (MOL) described by Liskovets.[12] This methodology has been successfully applied by Campo and Pérez[13] for the prediction of a complex problem related to laminar mixed convection in vertical pipes. These phenomena are governed by three strongly coupled partial differential equations.

With this background using the method of lines, our aim is to analyze the above-mentioned convective-radiative problem, wherein the radial discretization steps in the set of differential equations is performed by the control volume approach implemented by Patankar.[14] This combination gives rise to a new method of lines with control volumes (MOLCV). Results based on coarse grids having 10 lines unequally distributed along the pipe radius are presented for global quantities, such as the mean bulk temperature distribution and the total heat transfer rate.

## Problem formulation

Consider a fully developed turbulent gas flowing inside a circular pipe. At $x=0$, the gas temperature is uniform and equal to $T_e$ and for $x>0$ the outer surface of the pipe is maintained at $T_w$. To initiate the study, the participating gas is assumed to be gray, emitting, absorbing, and having constant thermophysical properties. In addition, viscous dissipation effects are considered negligible. Under these idealizations, the descriptive conservation equations in the thermal entrance region of the pipe can be written as

$$0=-\frac{1}{\rho}\frac{\mathrm{d}p}{\mathrm{d}x}+\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left[r(\nu+\nu_t)\frac{\mathrm{d}\bar{u}}{\mathrm{d}r}\right] \tag{1}$$

$$\bar{u}\frac{\partial T}{\partial x}=\frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\alpha+\frac{\nu_t}{\sigma_t}\right)\frac{\partial T}{\partial r}\right]-\frac{1}{\rho c_p}\operatorname{div}\mathbf{q}^r \tag{2}$$

### Nomenclature

| | |
|---|---|
| $A^+$ | van Driest's constant, $A^+=26$ |
| $c_p$ | Specific heat at constant pressure, J/kg K |
| $C_\tau, C_D$ | Constants of turbulence model: $C_\tau=0.548$, $C_D=0.164$ |
| $D$ | Tube diameter, m |
| $G$ | Total irradiation, W/m$^2$ sr |
| $G^*$ | Dimensionless total irradiation, Equation 5 |
| $K_a$ | Total volumetric absorption coefficient, m$^{-1}$ |
| $K$ | Turbulent kinetic energy, m$^2$/s$^2$ |
| $K^+$ | Dimensionless turbulent kinetic energy, $K^+=K/u_\tau^2$ |
| $k$ | Thermal conductivity, W/m K |
| $L$ | Turbulence length scale, m |
| $L^+$ | Dimensionless turbulence length scale, $L^+=Lu_\tau/\nu$ |
| $N$ | Radiation-conduction parameter, Equation 5 |
| $\mathrm{Nu}_T$ | Total Nusselt number, Equation 16 |
| Pr | Prandtl number, Equation 5 |
| $p$ | Pressure, Pa |
| $q^r$ | Radiation flux vector, W/m$^2$ |
| $q_w$ | Wall heat flux, W/m$^2$ |
| $R$ | Pipe radius, m |
| $r$ | Radial coordinate, m |
| $r^*$ | Dimensionless radial coordinate, Equation 5 |
| $r^+$ | Dimensionless wall radial coordinate, $r^+=ru_\tau/\nu$ |
| Re | Reynolds number |
| $T$ | Absolute temperature, K |
| $T_{ref}$ | Reference absolute temperature, K |
| $t$ | Dimensionless temperature, Equation 5 |
| $u$ | Axial velocity, m/s |
| $u_m$ | Mean velocity, m/s |
| $u^+$ | Dimensionless velocity, $u^+=u/u_\tau$ |
| $u_\tau$ | Shear velocity, $u_\tau=\tau_w/\rho$ |
| $x$ | Axial coordinate |
| $x^*$ | Dimensionless axial coordinate, Equation 5 |
| $y$ | Distance from wall, m |
| $y^+$ | Wall coordinate, $y^+=yu_\tau/\nu$ |

*Greek letters*

| | |
|---|---|
| $\alpha$ | Thermal diffusivity, m$^2$/s |
| $\beta$ | Extinction coefficient, m$^{-1}$ |
| $\varepsilon_w$ | Wall emissivity |
| $\rho$ | Density, kg/m$^3$ |
| $\nu$ | Kinematic viscosity, m$^2$/s |
| $\nu_t$ | Turbulent diffusivity, m$^2$/s |
| $\nu_t^+$ | Dimensionless turbulent diffusivity, $\nu_t^+=\nu_t/\nu$ |
| $\sigma$ | Stefan-Boltzmann constant, W/m$^2$ K$^4$ |
| $\sigma_t$ | Turbulent Prandtl number |
| $\tau_w$ | Wall shear stress, N/m$^2$ |
| $\tau_R$ | Optical thickness, Equation 5 |

*Subscripts*

| | |
|---|---|
| e | Entrance |
| b | Bulk |
| w | Wall |
| c | Conduction |
| r | Radiation |

where $\nu_t$ and $\sigma_T$ designate the turbulent diffusivity and the turbulent Prandtl number, respectively. In the above equations, the conventional notations are defined in the Nomenclature. The term $\operatorname{div}\mathbf{q}^r$, which corresponds to thermal radiation, is retained here in recognition of its importance in participating gas flows at high temperature. In the present study, the method of moments is adopted in order to determine an approximate solution of the radiation transport equation. Accordingly, the radiative heat flux vector $\mathbf{q}^r$ is expressed in terms of the total irradiation $G$, as follows:

$$\operatorname{div}\mathbf{q}^r = -K_a(G-4\sigma T^4) \tag{3}$$

where the governing equation for $G$ is

$$\frac{\partial^2 G}{\partial x^2}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial G}{\partial r}\right)=3\beta K_a(G-4\sigma T^4) \tag{4}$$

Details of these derivations are fund in Osizik.[9] At this juncture, it should be added that scattering has been neglected in the formulation of these equations. By introduction of the dimensionless quantities

$$\begin{aligned} &t=T/T_{ref}, \quad r^*=r/R, \quad x^*=x/R\mathrm{Pe}\\ &\mathrm{Pr}=\nu/\alpha, \quad \mathrm{Pe}=u_m D/\alpha\\ &N=\sigma R T_{ref}^3/k, \quad \tau_R=K_a R, \quad G^*=G/4\sigma T_{ref}^4 \end{aligned} \tag{5}$$

where $T_{ref}$ is defined as a reference temperature, Equations 2–4 are transformed to

$$\frac{1}{2}\frac{\bar{u}}{u_m}\frac{\partial t}{\partial x^*}=\frac{1}{r^*}\frac{\partial}{\partial r^*}\left[r^*\left(1+\frac{\mathrm{Pr}}{\sigma_t}\cdot\nu_t^+\right)\frac{\partial t}{\partial r^*}\right]+4N\tau_R(G^*-t^4) \tag{6}$$

$$\frac{1}{\mathrm{Pe}^2}\frac{\partial^2 G^*}{\partial x^{*2}}+\frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^*\frac{\partial G^*}{\partial r^*}\right)=3\tau_R^2(G^*-t^4) \tag{7}$$

Inspecting Equation 7 from physical grounds, in general, it may be added that for gas flows ($\mathrm{Pr}=0.7$) in small-to-moderate diameter pipes $\mathrm{Pe}\gg 1$. An approximate guideline for the threshold of axial thermal radiation has been reported by Pearce[17] and is reproduced in Appendix A. Consequently, the first term of the LHS of Equation 7 associated to the axial transport of thermal radiation may be discarded. This conclusion has been corroborated independently by Echigo *et al.*[15] and by Campo and Jarrin.[16] These authors solved the conjugate version of the laminar problem under study here, and showed that, under those circumstances axial radiation penetrates one or two diameters upstream of the origin of the heat exchange section. Undoubtedly, this effect is expected to be much smaller in turbulent flow regimes. Therefore, Equation 7 may be conveniently reduced to an ordinary differential equation. That is

$$\frac{1}{r^*}\frac{d}{dr^*}\left(r^*\frac{dG^*}{dr^*}\right)=3\tau_R^2(G^*-t^4) \tag{8}$$

Note that the same set of equations, i.e., Equations 6 and 8, may be also obtained employing the two-flux model for the intensity of radiation originally proposed by Milne-Eddington.[9]

Conversely, the relevant boundary conditions assigned to Equations 6 and 8 are expressed in dimensionless form as follows:

$$t=t_e, \quad x^*=0 \tag{9}$$

$$\frac{\partial t}{\partial r^*}=0, \quad r^*=0 \tag{10}$$

$$\frac{dG^*}{dr^*}=0, \quad r^*=0 \tag{11}$$

$$t=t_w, \quad r^*=1 \tag{12}$$

$$\frac{dG^*}{dr^*}=-\frac{3}{2}\left(\frac{\varepsilon_w}{2-\varepsilon_w}\right)\tau_R(G^*-t_w^4), \quad r^*=1 \tag{13}$$

The pipe-wall boundary conditions for $G^*$ given by Equation 13, controlling the radiative exchange at $r^*=1$, is examined in Appendix B.

The solution methodology for this set of equations will be delineated shortly. Meanwhile, the physical quantities of interest for practical applications of internal forced convection exposed to isothermal wall conditions are the mean bulk temperature distribution, $t_b(x^*)$,

$$t_b(x^*)=\frac{\int_0^1 t(x^*,r^*)u(r^*)r^*\,dr^*}{\int_0^1 u(r^*)r^*\,dr^*} \tag{14}$$

and the local surface heat flux $q_w(x^*)$,

$$q_w(x^*)=q_w^c(x^*)+q_w^r(x^*) \tag{15}$$

The components of the preceding equation are

$$q_w^c=-k\left.\frac{\partial T}{\partial r}\right|_{r=R} \tag{15a}$$

and

$$q_w^r=-\frac{1}{3K_a}\left.\frac{dG}{dr}\right|_{r=R} \tag{15b}$$

describing the local conductive and the local radiative transfer at the pipe-wall, respectively. These quantities lead to the traditional definition of the total Nusselt number

$$\mathrm{Nu}_T=\frac{2q_w R}{k(T_w-T_b)} \tag{16}$$

Hence, combining Equations 13, 15, and 16, one can write

$$\mathrm{Nu}_T=\frac{1}{t_w-t_b}\left[2\left.\frac{\partial t}{\partial r^*}\right|_{r^*=1}-4N\left(\frac{\varepsilon_w}{2-\varepsilon_w}\right)(G_w^*-t_w^4)\right] \tag{16a}$$

In this equation, the first term of the RHS corresponds to $q_w^c$ and has been calculated using a new finite difference formulation for the temperature gradient at the wall. This formulation is very accurate for coarse grids as those used in MOLCV and was developed in Ref. 18. Meanwhile, the second term of the RHS describes $q_w^r$ and has been evaluated in a direct way by the method of moments.

In light of the foregoing, it should be emphasized that calculation of the total Nusselt number $\mathrm{Nu}_T$ has been performed with the sole purpose of comparing our results against others published in the open literature. Nevertheless, the total heat transfer rate in a pipe of length $L$ may be easily determined from an energy balance between the stations $x=0$ and $x=L$. It relates the bulk temperature ratio $T_b/T_{ref}$ to a so-called heat transfer efficiency $\Omega$, defined as

$$\Omega=Q_T/Q_{ideal} \tag{16b}$$

Here $Q_{ideal}$ corresponds to the ideal heat transferred in an infinitely long pipe under identical thermal conditions. In view of this approach, the simple relation

$$\Omega=\frac{t_e-t_{bL}}{t_e-t_w} \tag{16c}$$

allows for a physical and direct calculation of the total heat transfer rate ($0<x<L$), once the mean bulk temperature distribution is known at $x=L$. Notice that the traditional approach to calculate the total heat transfer via the local Nusselt number is much more elaborate. It requires the numerical

integration of both distributions for the bulk temperature and for the local Nusselt number. The average values of these quantities are then introduced into the so-called Newton's equation of cooling.

## Model for turbulent transport

Calculation of the temperature distribution in the thermal entrance region requires a priori knowledge of the velocity and eddy-diffusivity profiles $\bar{u}$ and $v_t$, respectively in the gas flow. In this work, the $(K, L)$-model of turbulence is used, in which the distribution of $K$ is determined by solving the balance equation of turbulent kinetic energy, together with the equations associated to the mean flow. Some closure assumptions are made in modeling the terms appearing in these equations, to interrelate the variables $\bar{u}$, $K$, and $L$ only, the latter being prescribed algebraically. Accordingly, the turbulent kinetic energy equation for the situation considered here is given by Rodi:[11]

$$0=v_t^+\left(\frac{du^+}{dr^+}\right)^2-\frac{C_D K^{+3/2}}{L^+}+\frac{1}{r^+}\frac{d}{dr^+}\left(r^+\frac{v_t^+}{\sigma_K}\frac{dK^+}{dr^+}\right) \tag{17}$$

Moreover, on dimensional grounds, the eddy-diffusivity may be expressed in terms of $K$ and $L$ as

$$v_t^+=C_\tau K^{+1/2}L^+ \tag{18}$$

In addition to Equations 17 and 18, the $(K, L)$-model requires that $L^+$ be prescribed algebraically. Following the recommendation of Cebeci,[19] it may be evaluated from the relation

$$\frac{L^+}{R^+}=[0.14-0.08(1-y^+/R^+)^2-0.06(1-y^+/R^+)^4] \times[1-\exp(-y^+/A^+)] \tag{19}$$

Conversely, a first integration of the mean axial-momentum equation, Equation 1 for hydrodynamically developed conditions yields

$$\frac{du^+}{dr^+}=\frac{-r^+/R^+}{1+v_t^+}, \qquad u^+(R^+)=0 \tag{20}$$

At this stage, in order to reduce computational effort, integration of Equations 17 and 20 is started from the equilibrium region, say at $y=y_0$ and subject to the customary approximate boundary conditions. Accordingly, in the equilibrium region, the diffusion of turbulent kinetic energy is small compared to its production and dissipation, as seen in Equation 17. Thus, neglecting the diffusion term in this equation leads to the following expression for the turbulent kinetic energy in the near wall region:

$$\sqrt{K^+}=\frac{-C_D/L^+ + \sqrt{(C_D/L^+)^2+4(C_D C_\tau)^{1.5}(r^+/R^+)}}{2C_D C_\tau} \tag{21}$$

Computed velocity distributions using the proposed procedure for a wide variety of turbulent conditions were compared against solutions published by Rieke[20] and Jischa and Rieke.[21] In general, good agreement was obtained and some typical comparisons in terms of thermal parameters will be discussed in the next sections.

## Solution methodology

Once the formulation of the governing equations is completed, a suitable numerical solution technique must be developed. In view of this, it is important to know the level of accuracy of the adopted solution technique in order to assess the validity of the turbulence model when predictions of velocity are compared to experimental data. Additionally, it is also desirable that the technique gives reliable results with a reasonable computer time as a by-product. It is a well-known fact that finite difference procedures have proven to be useful in meeting the foregoing goals. Accordingly, the system of Equations 6 and 8 subject to the boundary conditions expressed by Equations 9–12, was solved numerically by a hybrid method combining the method of lines and the control volume approach (MOLCV).

The method of lines has been summarized by Liskovets[11] as a technique that replaces a partial differential equation in two independent variables by an appropriate system of ordinary differential equations. If the independent variables are $x^*$ and $r^*$ as in Equation 6, then, the region of integration may be divided into strips parallel to one of the coordinates by lines of $r^*=$constant. Accordingly, the participating partial derivatives with respect to $r^*$ need to be represented by finite difference formulations. The salient feature in the implementation of the method of lines in this work is that the discretization process in the radial direction is carried out by the control volume approach devised by Patankar[14] (see Figure 1). Correspondingly, this rather simple methodology generates a system of ordinary differential equations of first order, where the dependent variables are the temperatures along each line, in terms of the only independent variable: the axial coordinate $x^*$. Likewise, to be consistent with MOLCV, discretization of the ordinary differential equation controlling the irradiation $G^*$, Equation 8, has been performed using control volumes too. This procedure leads to an associate system of algebraic equations assigning a value of $G^*$ to each participating line.

## Variable grid spacing

The finite difference formulas derived above allow the use of a variable grid in the $r^*$ direction, which permits shorter steps close to the wall and longer steps away from it. The use of a nonuniform grid is particularly important in this problem in order to accommodate the characteristic steep temperature profiles in the vicinity of the wall caused by the combined action of turbulence and thermal radiation. The variable grid used here has the property such that the ratio

$$\ln\frac{y_u^*}{y_d^*}=\delta \tag{22}$$

is a constant, so that the distance to the $i$th grid line is

$$y_i=y_0\exp[(i-1)\delta] \tag{23}$$

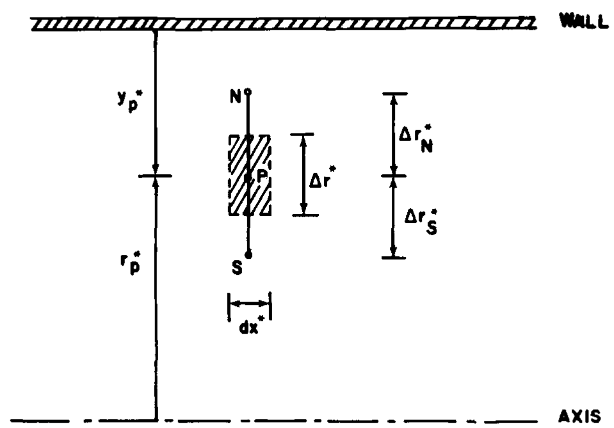


*Figure 1* Sketch for the transversal control volume

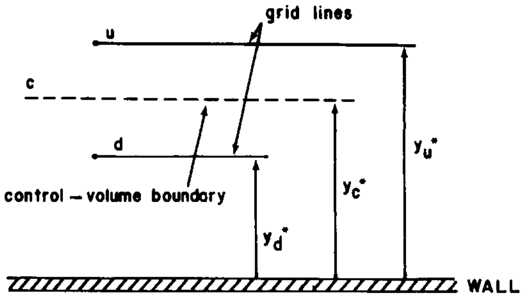


*Figure 2* Location of the control volume interfaces

where $y_0$ is the distance between the wall and the first grid line. Moreover, the control volume interfaces are placed in a way that the relationship:

$$\frac{\ln(y_c^*/y_d^*)}{\ln(y_u^*/y_d^*)}=\frac{1}{2} \tag{24}$$

holds (see Figure 2). In addition to this, from the wall inward, Equation 21 is used in conjunction with Equations 18 and 20 to provide the values of $u^+$ and $v_t^+$ through the viscous sublayer. Throughout the remainder of the flow domain, the $(K, L)$-model given by Equation 17 is utilized.

On the other hand, instead of using the customary linear formulation for the derivatives of the heat fluxes appearing in the discretized energy equation, Schuler and Campo[22] suggested the use of a log-law, which physically represents a better approximation for the temperature profile in bounded turbulent flows. This approach is also implemented here, and as a direct result it reduces the computational effort drastically. In light of the foregoing, a log-profile for the radial temperature of the form

$$t=a\ln(by^*) \tag{25}$$

is supposed to exist between adjacent grid lines $i$ and $i+1$, respectively, where the temperature is assumed to be given. After performing some algebraic manipulations, the following expression can be employed to evaluate the heat fluxes through all control volume interfaces

$$\left.\frac{\partial t}{\partial r^*}\right|_c=\frac{t_d-t_u}{\delta y_c^*} \tag{26}$$

Ultimately, the resulting coupled system of ordinary differential equations of first order and the system of algebraic equations were solved numerically by a Runge-Kutta-Fehlberg integration scheme in conjunction with the Thomas algorithm, respectively.

## Results and discussion

To establish the accuracy of the numerical methodology proposed here, MOLCV, predicted convective Nusselt numbers were compared with available benchmark solutions for turbulent pipe flow of a transparent gas (no radiation) with a Prandtl number of 0.7. First, results of the computed asymptotic Nusselt number covering a wide spectrum of Re from $10^4$ to $10^6$ are listed in Table 1, along with the results of Kays,[23] Gnielinski,[24] and Hallman *et al.*[25] Agreement seems to be satisfactory for all cases tested. Second, Figure 3 shows the computed Nusselt number in the thermal entrance region compared to the analytical solution of Kays[23] and the experiments of Hasegawa and Fujita[26] for Re = 50,000. Here again, both sets of results are found to be in good agreement with our predictions. The above-mentioned computations were based on a coarse grid having only ten lines nonuniformly distributed in the radial direction. The hybrid procedure developed in this paper proved to be stable, giving accurate results with a surprisingly small CPU time.

Attention is now focused on the situation wherein turbulent convection and thermal radiation act simultaneously in the gas flow. It is appropriate to compare the present results with those of Tsou and Kang,[5] where the conjugate problem in the fluid domain of the thermal entrance region was solved using Green functions. In this reference, results are given only for the distributions of centerline temperature and total Nusselt number in both the upstream and downstream regions. In this respect, Figure 4 was prepared to illustrate the distributions of centerline temperature for $Re=10^4$ and $N=5$ and 25, respectively. Both curves overlap with those reported in Ref. 5, except in the neighborhood of the origin for $N=25$ where minor deviations are found. Obviously, these deviations are of negligible value. It should be added that preheating is allowed in Ref. 5 due to the conjugate character of the mathematical formulation. Additionally, Figure 5 depicts the comparison between the corresponding total Nusselt number distributions for the same set of parameters employed in Figure 4. The $Nu_T$ values reported in Ref. 5 are within a few percent of the present values, but the deviations in the vicinity of $x/R=0$ are of opposite sign. The crossing of the curves in this region is caused by the allowance of preheating in Ref. 5.

**Table 1** Comparison of the asymptotic Nusselt number $Nu_\infty$ without radiation (Pr = 0.7)

| Re | Kays[23] | Gnielinski[24] | Hallman *et al.*[25] | Present work |
|---|---|---|---|---|
| 10,000 | 29.2 | 27.6 | 32.3 | 29.3 |
| 20,000 | 50.8 | 49.3 | 52.3 | 50 |
| 50,000 | 105 | 105 | 102 | 103 |
| 100,000 | 184 | 186 | 172 | 178 |
| 1,000,000 | 1161 | 1169 | — | 1160 |

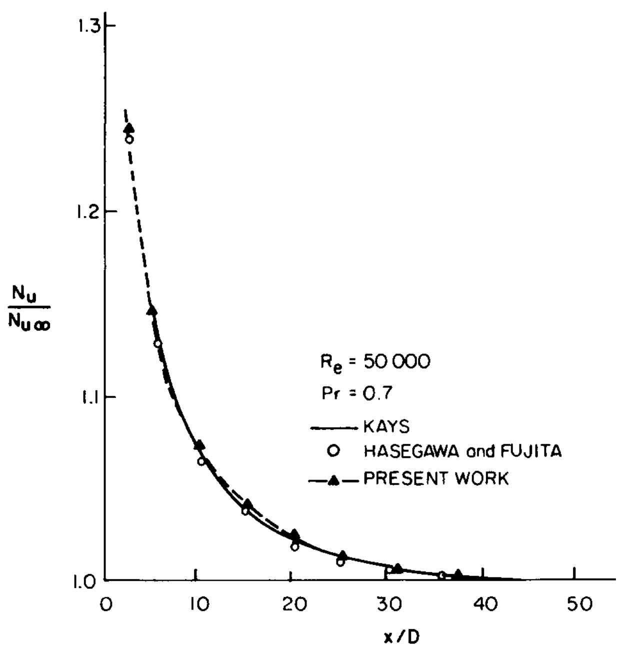


*Figure 3* Comparison of the Nusselt number distribution in the absence of radiation

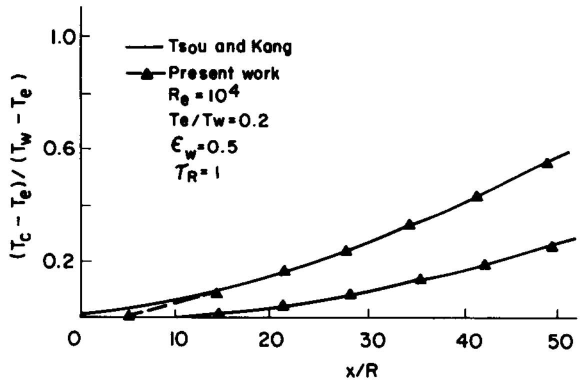


*Figure 4* Comparison of the centerline temperature distribution

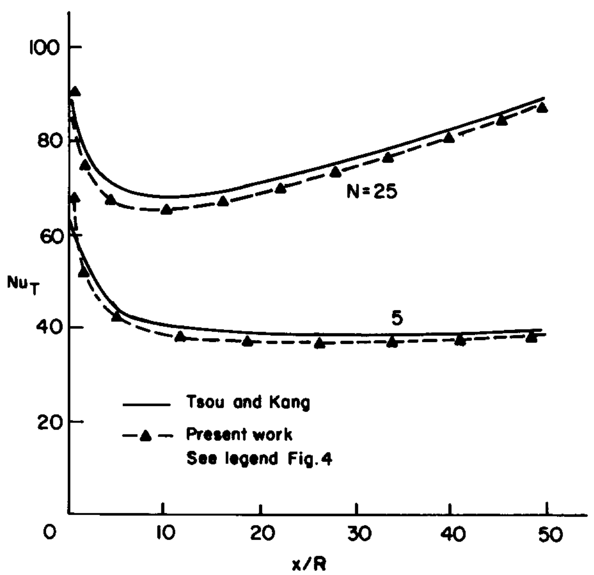


*Figure 5* Comparison of the total Nusselt number distribution

Inspection of the downstream portion of Figure 5 shows that the curves tend to attain a shallow minimum and then rise gradually. The location of these minima depends on the relative importance of the radiation mechanism compared to its conduction counterpart. Interestingly enough, for the largest $x/R$ of the graph for weak radiation ($N=5$) the rise is very small—just a few percent. On the other hand, the presence of strong radiation ($N=25$) does not lend to the just mentioned behavior. Therefore, for strong radiation, the minimum Nusselt number is shifted toward the inlet and for no radiation the minimum Nusselt number coincides with the asymptotic value.

As a side comment, it should be added that although the isothermal boundary condition gives rise to a thermally developed regime in which Nu is independent of $x$, this is not the case when strong radiation interacts with convection in the gas flow. Consequently, use of an asymptotic Nusselt number in the presence of strong radiation is physically unrealistic and is of little importance, other than to compare results between different computational procedures.

One of the main objectives of this paper is to stress the fact that the quantity of most direct practical interest in internal flows is the mean bulk temperature distribution and its relationship to the total heat transferred as stated in Equation 16c. Accordingly, the results for the mean bulk temperature distribution $t_b(x^*)$ are presented in Figures 6–9 for various representative convective and radiative parameters. General trends will be discussed here in detail. The initial discussion will be focused on Figure 6 which shows the influence of the radiation-conduction parameter $N$ on the temperature development for fixed values of $Re = 5 \times 10^5$ and $\tau_R = 1$. Here, it is seen that an increase in $N$ causes a rapid increase in $t_b$ at a given axial station $x^*$. This behavior is due to the fact that, with increased $N$, the difference between the wall and fluid temperatures is more pronounced and thus radiative interaction between them becomes higher. As a consequence of this, the thermal entry length tends to decrease as $N$ increases.

To show the influence of the Reynolds number on combined heat transfer rates, results are presented in Figure 7 for the same radiation parameters as in the previous Figure 6, only changing the Reynolds number to $Re = 10^5$. As expected, mean bulk temperature development is increased with increasing Reynolds number as a direct result of the increased value of eddy-diffusivity. In addition, the relative importance of radiation is decreased somewhat due to the faster development of temperature. For this case, it can be observed that the deviations between the curves of $N=0$ and $N=10$ are small enough to neglect the radiative transfer contribution for engineering

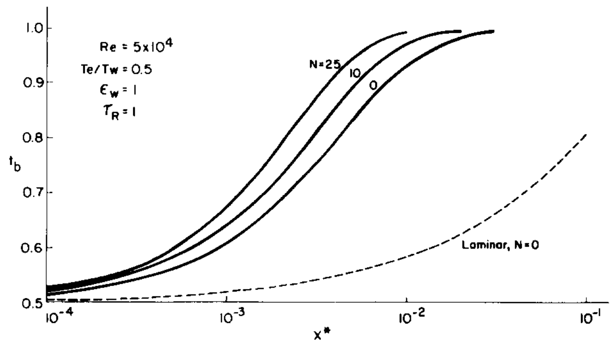


*Figure 6* Influence of $N$ on the mean bulk temperature distributions

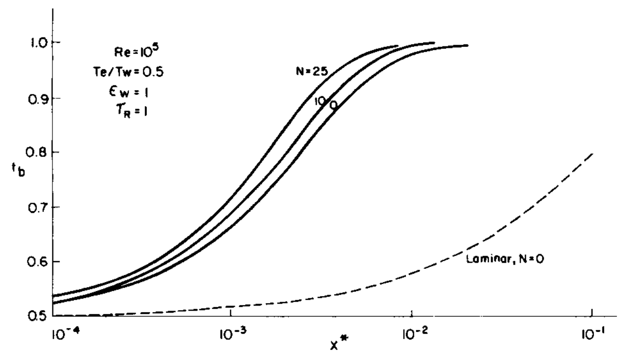


*Figure 7* Influence of $N$ on the mean bulk temperature distributions

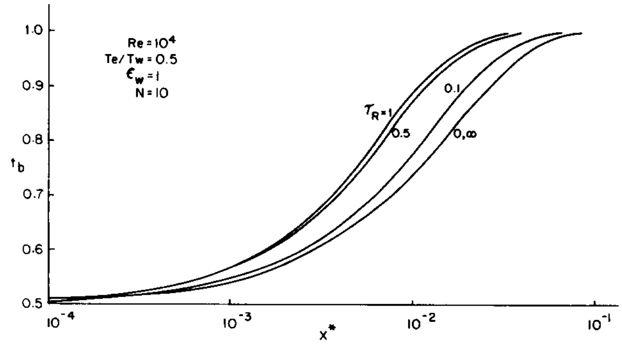


*Figure 8* Effects of $\tau_R$ on the mean bulk temperature distributions

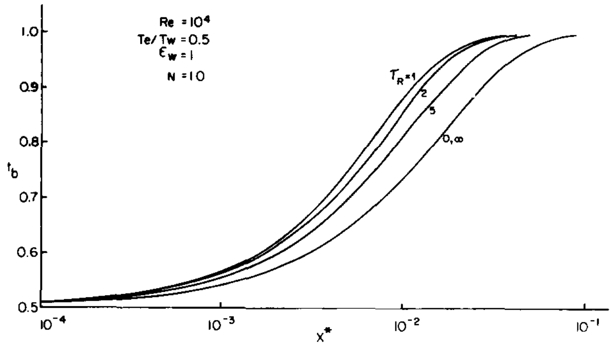


*Figure 9* Effects of $\tau_R$ on the mean bulk temperature distributions

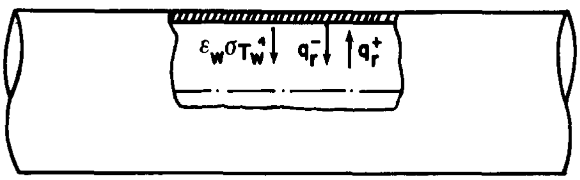

*Figure 10* Radiative energy balance at the wall

applications. However, this is not true for the case of $Re=4\times10^5$ where the corresponding curves are wide apart as can be noticed in Figure 6.

Attention may now be turned to Figures 9 and 10 wherein the heat transfer enhancement characteristics of the gas flow will be discussed. Within this framework, the optical thickness of the gas has a definite physical meaning. In order to explore this, a set of parameters has been kept fixed as stated in the legends of these figures. From physical reasoning, it may be concluded that in the optically thin limit ($\tau_R\to0$) where neither absorption nor emission takes place in the medium, the radiative contribution must vanish. Similarly, for the optically thick limit ($\tau_R\to\infty$), thermal radiation is absorbed right at the point of its emission and the radiation contribution must vanish too. Therefore, as a result of this analysis, there should exist an optimum optical thickness for which maximum heat transfer rates are achieved. This particular phenomenon is clearly demonstrated in Figures 9 and 10 for thin and thick gases, respectively. After testing several values of $\tau_R$, maximum distributions of mean bulk temperature are obtained for $\tau_R=1$. This finding is manifested for other values of Re and is a clear testimony of a heat transfer enhancement.

## Conclusions

In this paper, the combination of turbulent forced convection and thermal radiation has been examined for the flow of an absorbing-emitting gas in an isothermal pipe. The significant role played by radiation in the thermal entry region has been successfully represented by the method of moments. Meanwhile, the turbulent transport of the gas flow has been described by the $(K, L)$-model. An explicit finite difference procedure for the prediction of turbulent radiating gas flows has been developed. The procedure takes advantage of the method of lines and the control volume discretization in the radial direction (MOLCV). It differs from the other methods in its directness, simplicity and great savings in computation time even with a coarse grid, and no lengthy iterative steps are required. The predictions agree well with other more elaborate numerical results published in the open literature for several situations involving turbulent flows. The success of the proposed MOLCV methodology appears sufficiently encouraging to justify further work in which the gas will be modeled as nongray.

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## Appendix A: Examination of axial thermal radiation

In order to obtain an *a priori* estimate of the conditions for which the axial transport of thermal radiation can be neglected, an order of magnitude argument similar to that used in boundary layer theory can be made. In fact, such an argument is given by Pearce[17] for laminar flow through circular tubes. Following his reasoning, which is based on the observation that the radiation flux in the optically thick limit will yield the most conservative criterion, the requirement is that

$$\rho c_p u \frac{\partial T}{\partial x} \gg \frac{\partial}{\partial x} \mathbf{q}^{\mathrm{r}} x \tag{A1}$$

This inequality is a statement that the contribution of the axial component of the radiation heat flux is very much smaller than that of the axial convection term. In the optically thick limit, the radiation flux is given by

$$\mathbf{q}^{\mathrm{r}} x = -\frac{16\sigma T^3}{3K_{\mathrm{a}}} \frac{\partial T}{\partial x} \tag{A2}$$

This criterion, when written in the context of the present problem, becomes

$$\frac{3\rho u c_p K_{\mathrm{a}} R}{16\sigma T_{\mathrm{ref}}^3} \gg 1 \tag{A3}$$

Equivalently, the radiation Peclet number $\mathrm{Re}\,\mathrm{Pr}(\tau_{\mathrm{R}}/N)$ is restricted by the inequality

$$\mathrm{Re}\,\mathrm{Pr}(\tau_{\mathrm{R}}/N) \gg 10 \tag{A4}$$

In this sense, condition (A4) is analogous to $\mathrm{Re}\,\mathrm{Pr} \gg 1$ for the neglect of the axial temperature gradient in purely forced convection pipe flows.

## Appendix B: Boundary condition for irradiation at the pipe wall

Performing a radiative energy balance at the wall shown in Figure 10 gives the relation

$$q_{\mathrm{r}}^- = (1-\varepsilon_{\mathrm{w}})q_{\mathrm{r}}^+ + \varepsilon_{\mathrm{w}}\sigma T_{\mathrm{w}}^4 \tag{B1}$$

On the other hand, $q_{\mathrm{r}}^+$ and $q_{\mathrm{r}}^-$ are defined, via the method of moments, by

$$q_{\mathrm{r}}^+ = \int_{\Omega\hat{e}_{\mathrm{r}}>0} \mathbf{\Omega}\cdot\hat{e}_{\mathrm{r}} I\,\mathrm{d}\omega = \frac{G}{4} - \frac{1}{6K_{\mathrm{a}}}\frac{\partial G}{\partial r} \tag{B2}$$

and

$$q_{\mathrm{r}}^- = \int_{\Omega\hat{e}_{\mathrm{r}}<0} \mathbf{\Omega}\cdot\hat{e}_{\mathrm{r}} I\,\mathrm{d}\omega = \frac{G}{4} + \frac{1}{6K_{\mathrm{a}}}\frac{\partial G}{\partial r} \tag{B3}$$

respectively, where the required integrations are carried out with help of Equation 3.

Next, combining Equations B1–B3 leads to the appropriate boundary condition, which written in dimensionless form, results in

$$\frac{\mathrm{d}G^*}{\mathrm{d}r^*} = -\frac{3}{2}\left(\frac{\varepsilon_{\mathrm{w}}}{2-\varepsilon_{\mathrm{w}}}\right)\tau_{\mathrm{R}}(G^* - t_{\mathrm{w}}^4), \qquad r^* = 1 \tag{B4}$$